A Class of Ratio Estimators of a Finite Population Mean Using Two Auxiliary Variables

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Abstract

In sample surveys, it is usual to increase the efficiency of the estimators by the use of the auxiliary information. We propose a class of ratio estimators of a finite population mean using two auxiliary variables and obtain mean square error (MSE) equations for the class of proposed estimators. We find theoretical conditions that make proposed family estimators more efficient than the traditional ratio estimator and the estimators proposed by Abu-Dayeh *et al.* using two auxiliary variables. In addition, we support these theoretical results with the aid of a numerical example.

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Introduction

Use of auxiliary information has been in practice to increase the efficiency of the estimators. Such information is generally used in ratio, product and regression type estimators for the estimation of population mean of study variable. When correlation between study variable and auxiliary variable is positive ratio method of estimation is used. On the other hand if the correlation is negative, product method of estimation is preferred. Some research works have been done in ratio, product and regression type estimators by using an auxiliary variable [1–10].

In this study, a class of ratio estimators using two auxiliary variables is considered to estimate a finite population mean for the variable of interest. We considered several special estimators of the suggested estimators. The comparisons between the traditional multivariate ratio estimators and the estimators proposed by Abu-Dayeh *et al.* [11] with the proposed family of estimators using information of two variables are considered. We compared the traditional ratio estimator, the estimators proposed by Abu-Dayeh *et al.* and proposed several special estimators using the statistic data given in Table 1. And we obtained the satisfactory results.

Materials and Methods

The Existed Estimators

The traditional multivariate ratio estimator using information of two auxiliary variables x_1 and x_2 to estimate the population mean, $\bar{Y}$, as follows:

$$\bar{y}_{MR}=\theta_1\bar{y}\frac{\bar{X}_1}{\bar{x}_1}+\theta_2\bar{y}\frac{\bar{X}_2}{\bar{x}_2} \tag{1}$$

where $\bar{x}_i$ and $\bar{X}_i (i=1,2)$ denote respectively the sample and the population means of the variable x_i; and θ_1,θ_2 are the weights that satisfy the condition: $\theta_1+\theta_2=1$ [12].

The MSE of this estimator is given by

$$MSE(\bar{y}_{MR})\cong\frac{1-f}{n}\bar{Y}^2(C_y^2+\theta_1^2C_{x_1}^2+\theta_2^2C_{x_2}^2 -2\theta_1\rho_{yx_1}C_yC_{x_1}-2\theta_2\rho_{yx_2}C_yC_{x_2}+2\theta_1\theta_2\rho_{x_1x_2}C_{x_1}C_{x_2}) \tag{2}$$

where C_y,C_{x_1} and C_{x_2} denote the coefficient of variation of Y, X_1 and X_2 respectively and $\rho_{yx_1},\rho_{yx_2},\rho_{x_1x_2}$ denote the correlation coefficient between Y and X_1, Y and X_2, X_1 and X_2 respectively.

The optimum values of θ_1 and θ_2 are given by

$$\theta_1^*=\frac{C_{x_2}^2-\rho_{yx_2}C_yC_{x_2}+\rho_{yx_1}C_yC_{x_1}-\rho_{x_1x_2}C_{x_1}C_{x_2}}{C_{x_1}^2+C_{x_2}^2-2\rho_{x_1x_2}C_{x_1}C_{x_2}},\ \theta_2^*=1-\theta_1^*$$

$$MSE_{\min}(\bar{y}_{MR})\cong\frac{1-f}{n}\bar{Y}^2(C_y^2+\theta_1^{*2}C_{x_1}^2+\theta_2^{*2}c_{x_2}^2 -2\theta_1^*\rho_{yx_1}C_yC_{x_1}-2\theta_2^*\rho_{yx_2}C_yC_{x_2}+2\theta_1^*\theta_2^*\rho_{x_1x_2}C_{x_1}C_{x_2}) \tag{3}$$

Abu-Dayeh *et al.* proposed the estimators using two auxiliary variables given by

$$\bar{y}_{r2}^{\gamma}=\bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\gamma_1}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \tag{4}$$

$$\bar{y}_{r2}^{\varepsilon}=\varepsilon_1\bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\gamma_1}+\varepsilon_2\bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \tag{5}$$

where $\varepsilon_1+\varepsilon_2=1$.

Table 1. Data Statistics.

$N=180$	$n=70$	$f=0.3889$	$\bar{Y}=13.9951$
$\bar{X}_1=27.3981$	$\bar{X}_2=38.7167$	$C_{x_1}=0.4254$	$C_{x_2}=0.3339$
$C_y=0.4180$	$\rho_{yx_1}=0.5630$	$\rho_{yx_2}=0.5273$	$\rho_{x_1x_2}=0.2589$
$\beta_2(x_1)=4.2724$	$\beta_2(x_2)=2.1546$		

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MSE of these estimators are given as follows:

$$MSE\left(\bar{y}_{r2}^{\gamma}\right) \cong \frac{1-f}{n}\overline{Y}^2\left[C_y^2+\gamma_1^2C_{x_1}^2+\gamma_2^2C_{x_2}^2+2\gamma_1\rho_{yx_1}C_yC_{x_1}+2\gamma_2\rho_{yx_2}C_yC_{x_2}+2\gamma_1\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right] \quad (6)$$

$$MSE\left(\bar{y}_{r2}^{\varepsilon}\right) \cong \frac{1-f}{n}\overline{Y}^2[C_y^2+\varepsilon_1^2\gamma_1^2C_{x_1}^2+\varepsilon_2^2\gamma_2^2C_{x_2}^2+2\varepsilon_1\gamma_1\rho_{yx_1}C_yC_{x_1}+2\varepsilon_2\gamma_2\rho_{yx_2}C_yC_{x_2}+2\varepsilon_1\gamma_2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2}] \quad (7)$$

The optimum values of γ_1 and γ_2 are given by

$$\gamma_1^*=\frac{C_y(\rho_{yx_2}\rho_{x_1x_2}-\rho_{yx_1})}{C_{x_1}(1-\rho_{x_1x_2}^2)},\quad \gamma_2^*=\frac{C_y(\rho_{yx_1}\rho_{x_1x_2}-\rho_{yx_2})}{C_{x_2}(1-\rho_{x_1x_2}^2)}$$

$$MSE_{\min}\left(\bar{y}_{r2}^{\gamma}\right) \cong \frac{1-f}{n}\ \bar{Y}^2C_y^2\left(1-\frac{\rho_{yx_1}^2+\rho_{yx_2}^2-2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}\right) \quad (8)$$

The optimum values of ε_1 and ε_2 are given by

$$\varepsilon_1^*=\frac{\gamma_2^2C_{x_2}^2-\gamma_1\rho_{yx_1}C_yC_{x_1}+\gamma_2\rho_{yx_2}C_yC_{x_2}-\gamma_1\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2}}{\gamma_1^2C_{x_1}^2-2\gamma_1\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2}+\gamma_2^2C_{x_2}^2},$$

$$\varepsilon_2^*=1-\varepsilon_1^*.$$

$$MSE_{\min}\left(\bar{y}_{r2}^{\varepsilon}\right) \cong \frac{1-f}{n}\ \bar{Y}^2[C_y^2+\varepsilon_1^{*2}\gamma_1^2C_{x_1}^2+\varepsilon_2^{*2}\gamma_2^2C_{x_2}^2+2\varepsilon_1^*\gamma_1\rho_{yx_1}C_yC_{x_1}+2\varepsilon_2^*\gamma_2\rho_{yx_2}C_yC_{x_2}+2\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2}] \quad (9)$$

The Proposed Family of Ratio Estimators

We propose a class of multivariate ratio estimators using information of two auxiliary variables as follows:

$$\bar{y}_{pmr}=w_1\bar{y}\frac{a_1\overline{X}_1+b_1}{a_1\bar{x}_1+b_1}+w_2\bar{y}\frac{a_2\overline{X}_2+b_2}{a_2\bar{x}_2+b_2} \quad (10)$$

where w_1 and w_2 are weights that satisfy the condition: $w_1+w_2=1$, $a_1(\neq 0), a_2(\neq 0), b_1, b_2$ are either real numbers or functions of known parameters.

MSE of these estimators can be found using Taylor series method defined as

$$f(\bar{y},\bar{x}_1,\bar{x}_2)\cong f(\overline{Y},\overline{X}_1,\overline{X}_2)+\left.\frac{\partial f}{\partial \bar{y}}\right|_{(\overline{Y},\overline{X}_1,\overline{X}_2)}(\bar{y}-\overline{Y})+\left.\frac{\partial f}{\partial \bar{x}_1}\right|_{(\overline{Y},\overline{X}_1,\overline{X}_2)}(\bar{x}_1-\overline{X}_1)+\left.\frac{\partial f}{\partial \bar{x}_2}\right|_{(\overline{Y},\overline{X}_1,\overline{X}_2)}(\bar{x}_2-\overline{X}_2) \quad (11)$$

where $f(\bar{y},\bar{x}_1,\bar{x}_2)=\bar{y}_{pmr}$

$$\bar{y}_{pmr}-\overline{Y}\cong(\bar{y}-\overline{Y})-w_1\frac{a_1\overline{Y}}{a_1\overline{X}_1+b_1}(\bar{x}_1-\overline{X}_1)-w_2\frac{a_2\overline{Y}}{a_2\overline{X}_2+b_2}(\bar{x}_2-\overline{X}_2)$$

$$=(\bar{y}-\overline{Y})-w_1\beta_1(\bar{x}_1-\overline{X}_1)-w_2\beta_2(\bar{x}_2-\overline{X}_2)$$

where $\beta_1=\dfrac{a_1\bar{Y}}{a_1\bar{X}_1+b_1}$, $\beta_2=\dfrac{a_2\bar{Y}}{a_2\bar{X}_2+b_2}$

MSE of the class of estimators are given as follows:

$$\begin{aligned}MSE(\bar{y}_{pmr})&=E(\bar{y}_{pmr}-\overline{Y})^2\\&\cong E[(\bar{y}-\overline{Y})^2+w_1^2\beta_1^2(\bar{x}_1-\overline{X}_1)^2+w_2^2\beta_2^2(\bar{x}_2-\overline{X}_2)^2\\&-2w_1\beta_1(\bar{y}-\overline{Y})(\bar{x}_1-\overline{X}_1)-2w_2\beta_2(\bar{y}-\overline{Y})(\bar{x}_2-\overline{X}_2)\\&+2w_1\beta_1w_2\beta_2(\bar{x}_1-\overline{X}_1)(\bar{x}_2-\overline{X}_2)]\end{aligned}$$

$$=\frac{1-f}{n}\overline{Y}^2[C_y^2+w_1^2\alpha_1^2C_{x_1}^2+w_2^2\alpha_2^2C_{x_2}^2-2w_1\alpha_1\rho_{yx_1}C_yC_{x_1}-2w_2\alpha_2\rho_{yx_2}C_yC_{x_2}+2w_1w_2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2}] \quad (12)$$

where $\alpha_1=\dfrac{a_1\overline{X_1}}{a_1\bar{X}_1+b_1}$, $\alpha_2=\dfrac{a_2\overline{X_2}}{a_2\bar{X}_2+b_2}$

The optimal values of w_1 and w_2 to minimize (12) can easily be found as follows:

$$w_1^*=\frac{\alpha_2^2C_{x_2}^2+\alpha_1\rho_{yx_1}C_yC_{x_1}-\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2}-\alpha_2\rho_{yx_2}C_yC_{x_2}}{\alpha_1^2C_{x_1}^2-2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2}+\alpha_2^2C_{x_2}^2},\ w_2^*=1-w_1^*$$

$$MSE_{\min}\left(\bar{y}_{pmr}\right)\cong\frac{1-f}{n}\ \bar{Y}^2[C_y^2+w_1^{*2}\alpha_1^2C_{x_1}^2+w_2^{*2}\alpha_2^2C_{x_2}^2-2w_1^*\alpha_1\rho_{yx_1}C_yC_{x_1}-2w_2^*\alpha_2\rho_{yx_2}C_yC_{x_2}+2w_1^*w_2^*\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2}] \quad (13)$$

Some Members of the Proposed Class of Ratio Estimators $\bar{y}_{pmr}$

The following are the proposed class of ratio estimators $\bar{y}_{pmr}$:

$$\bar{y}_{pmr}^{(0)} = \bar{y}_{MR} = w_1\bar{y}\frac{\bar{X}_1}{\bar{x}_1} + w_2\bar{y}\frac{\bar{X}_2}{\bar{x}_2} \text{ (The traditional ratio estimator)}$$

$$\bar{y}_{pmr}^{(1)} = w_1\bar{y}\frac{\overline{X}_1 + C_{x_1}}{\bar{x}_1 + C_{x_1}} + w_2\bar{y}\frac{\overline{X}_2 + C_{x_2}}{\bar{x}_2 + C_{x_2}}$$

$$\bar{y}_{pmr}^{(2)} = w_1\bar{y}\frac{\overline{X}_1 + \beta_2(x_1)}{\bar{x}_1 + \beta_2(x_1)} + w_2\bar{y}\frac{\overline{X}_2 + \beta_2(x_2)}{\bar{x}_2 + \beta_2(x_2)}$$

$$\bar{y}_{pmr}^{(3)} = w_1\bar{y}\frac{\overline{X}_1\beta_2(x_1) + C_{x_1}}{\bar{x}_1\beta_2(x_1) + C_{x_1}} + w_2\bar{y}\frac{\overline{X}_2\beta_2(x_2) + C_{x_2}}{\bar{x}_2\beta_2(x_2) + C_{x_2}}$$

$$\bar{y}_{pmr}^{(4)} = w_1\bar{y}\frac{\overline{X}_1 C_{x_1} + \beta_2(x_1)}{\bar{x}_1 C_{x_1} + \beta_2(x_1)} + w_2\bar{y}\frac{\overline{X}_2 C_{x_2} + \beta_2(x_2)}{\bar{x}_2 C_{x_2} + \beta_2(x_2)}$$

$$\bar{y}_{pmr}^{(5)} = w_1\bar{y}\frac{\overline{X}_1 + \rho_{yx_1}}{\bar{x}_1 + \rho_{yx_1}} + w_2\bar{y}\frac{\overline{X}_2 + \rho_{yx_2}}{\bar{x}_2 + \rho_{yx_2}}$$

$$\bar{y}_{pmr}^{(6)} = w_1\bar{y}\frac{\overline{X}_1 C_{x_1} + \rho_{yx_1}}{\bar{x}_1 C_{x_1} + \rho_{yx_1}} + w_2\bar{y}\frac{\overline{X}_2 C_{x_2} + \rho_{yx_2}}{\bar{x}_2 C_{x_2} + \rho_{yx_2}}$$

$$\bar{y}_{pmr}^{(7)} = w_1\bar{y}\frac{\overline{X}_1\rho_{yx_1} + C_{x_1}}{\bar{x}_1\rho_{yx_1} + C_{x_1}} + w_2\bar{y}\frac{\overline{X}_2\rho_{yx_2} + C_{x_2}}{\bar{x}_2\rho_{yx_2} + C_{x_2}}$$

$$\bar{y}_{pmr}^{(8)} = w_1\bar{y}\frac{\overline{X}_1\beta_2(x_1) + \rho_{yx_1}}{\bar{x}_1\beta_2(x_1) + \rho_{yx_1}} + w_2\bar{y}\frac{\overline{X}_2\beta_2(x_2) + \rho_{yx_2}}{\bar{x}_2\beta_2(x_2) + \rho_{yx_2}}$$

$$\bar{y}_{pmr}^{(9)} = w_1\bar{y}\frac{\overline{X}_1\rho_{yx_1} + \beta_2(x_1)}{\bar{x}_1\rho_{yx_1} + \beta_2(x_1)} + w_2\bar{y}\frac{\overline{X}_2\rho_{yx_2} + \beta_2(x_2)}{\bar{x}_2\rho_{yx_2} + \beta_2(x_2)}$$

The suitable choices of constants a_1,b_1,a_2 and b_2 are given in Table 2.

Table 2. The suitable choices of constants a_1,b_1,a_2 and b_2.

Estimators	a_1	b_1	a_2	b_2
$\bar{y}_{pmr}^{(0)}$	1	0	1	0
$\bar{y}_{pmr}^{(1)}$	1	C_{x_1}	1	C_{x_2}
$\bar{y}_{pmr}^{(2)}$	1	$\beta_2(x_1)$	1	$\beta_2(x_2)$
$\bar{y}_{pmr}^{(3)}$	$\beta_2(x_1)$	C_{x_1}	$\beta_2(x_2)$	C_{x_2}
$\bar{y}_{pmr}^{(4)}$	C_{x_1}	$\beta_2(x_1)$	C_{x_2}	$\beta_2(x_2)$
$\bar{y}_{pmr}^{(5)}$	1	ρ_{yx_1}	1	ρ_{yx_2}
$\bar{y}_{pmr}^{(6)}$	C_{x_1}	ρ_{yx_1}	C_{x_2}	ρ_{yx_2}
$\bar{y}_{pmr}^{(7)}$	ρ_{yx_1}	C_{x_1}	ρ_{yx_2}	C_{x_2}
$\bar{y}_{pmr}^{(8)}$	$\beta_2(x_1)$	ρ_{yx_1}	$\beta_2(x_2)$	ρ_{yx_2}
$\bar{y}_{pmr}^{(9)}$	ρ_{yx_1}	$\beta_2(x_1)$	ρ_{yx_2}	$\beta_2(x_2)$

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Efficiency Comparisons

We compare the MSE of the proposed class of ratio estimators given in Eq. (13) with the MSE of the traditional ratio estimator given in Eq.(3)as follows:

$$\begin{aligned} &MSE_{\min}(\bar{y}_{pmr}) < MSE_{\min}(\bar{y}_{MR}) \Leftrightarrow \\ &(w_1^{*2}\alpha_1^2 - \theta_1^{*2})C_{x_1}^2 + (w_2^{*2}\alpha_2^2 - \theta_2^{*2})C_{x_2}^2 \\ &-2(w_1^*\alpha_1 - \theta_1^*)\rho_{yx_1}C_yC_{x_1} - 2(w_2^*\alpha_2 - \theta_2^*)\rho_{yx_2}C_yC_{x_2} \\ &+2(w_1^*w_2^*\alpha_1\alpha_2 - \theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2} < 0 \end{aligned} \tag{14}$$

When this condition is satisfied, the proposed class of ratio estimators $\bar{y}_{pmr}$ will be more efficient than the traditional ratio estimator.

We compare the MSE of the proposed class of ratio estimators given in Eq. (13) with the MSE of the estimators proposed by Abu-Dayeh *et al.* given in Eq. (8) and Eq. (9) as follows:

$$\begin{aligned} &MSE_{\min}(\bar{y}_{pmr}) < MSE_{\min}\left(\bar{y}_{r2}^{\gamma}\right) \Leftrightarrow \\ &w_1^{*2}\alpha_1^2C_{x_1}^2 + w_2^{*2}\alpha_2^2C_{x_2}^2 - 2w_1^*\alpha_1\rho_{yx_1}C_yC_{x_1} \\ &-2w_2^*\alpha_2\rho_{yx_2}C_yC_{x_2} + 2w_1^*w_2^*\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} \\ &< -C_y^2\frac{\rho_{yx_1}^2 + \rho_{yx_2}^2 - 2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2} \end{aligned} \tag{15}$$

$$\begin{aligned} &MSE_{\min}(\bar{y}_{pmr}) < MSE_{\min}\left(\bar{y}_{r2}^{\varepsilon}\right) \\ &(w_1^{*2}\alpha_1^2 - \varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2 + (w_2^{*2}\alpha_2^2 - \varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2 \\ &-2(w_1^*\alpha_1 + \varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1} - 2(w_2^*\alpha_2 + \varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2} \\ &+2(w_1^*w_2^*\alpha_1\alpha_2 - \varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2} < 0 \end{aligned} \tag{16}$$

When these conditions are satisfied, the proposed class of ratio estimators $\bar{y}_{pmr}$ will be more efficient than the estimators proposed by Abu-Dayeh *et al.*

Numerical Illustration

In this section, we apply the traditional ratio estimator, the estimators proposed by Abu-Dayeh *et al.* and some members of the proposed class of estimators $\bar{y}_{pmr}$, to data whose statistics are given in Table 1 [13]. We assume to take the sample size $n = 70$, from $N = 180$ using SRSWOR. The MSE of these estimators are computed.

Table 3. MSE Values of Ratio Estimators.

Estimators	MSE
$\bar{y}_{MR}$	0.157645
$\bar{y}_{r2}^{\gamma}$	0.157421
$\bar{y}_{r2}^{\varepsilon}$	0.192279 $(\gamma_1=\gamma_1^*,\gamma_2=\gamma_2^*)$
$\bar{y}_{pmr}^{(1)}$	0.157526
$\bar{y}_{pmr}^{(2)}$	0.157911
$\bar{y}_{pmr}^{(3)}$	0.157601
$\bar{y}_{pmr}^{(4)}$	0.162033
$\bar{y}_{pmr}^{(5)}$	0.157489
$\bar{y}_{pmr}^{(6)}$	0.157427
$\bar{y}_{pmr}^{(7)}$	0.157463
$\bar{y}_{pmr}^{(8)}$	0.157581
$\bar{y}_{pmr}^{(9)}$	0.159698
$\bar{y}_{pmr}^{(10)}$	0.157421 $(a_1=0.43,\ b_1=0.78,\ a_2=1.22,\ b_2=0.75)$

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Results and Discussion

MSE of the traditional ratio estimator $\bar{y}_{MR}$, the estimators $\bar{y}_{r2}^{\gamma}, \bar{y}_{r2}^{\varepsilon}$ proposed by Abu-Dayeh *et al.* and some members of the proposed ratio estimators $\bar{y}_{pmr}$ can be seen in Table 3.

From Table 3, we understand that the proposed ratio estimators $\bar{y}_{pmr}^{(1)}$, $\bar{y}_{pmr}^{(3)}$, $\bar{y}_{pmr}^{(5)}$, $\bar{y}_{pmr}^{(6)}$, $\bar{y}_{pmr}^{(7)}$, $\bar{y}_{pmr}^{(8)}$ and $\bar{y}_{pmr}^{(10)}$ are more efficient than the traditional ratio estimator using two auxiliary variables. When we examine the condition (14), for this data set, we see that all of them are satisfied as follows:

(1) the proposed ratio estimator $\bar{y}_{pmr}^{(1)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-6.9469\times10^{-5}<0\end{aligned}$$

(2) the proposed ratio estimator $\bar{y}_{pmr}^{(3)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-2.6063\times10^{-5}<0\end{aligned}$$

(3) the proposed estimator $\bar{y}_{pmr}^{(5)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-9.1073\times10^{-5}<0\end{aligned}$$

(4) the proposed ratio estimator $\bar{y}_{pmr}^{(6)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.00012785<0\end{aligned}$$

(5) the proposed ratio estimator $\bar{y}_{pmr}^{(7)}$.

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.00010669<0\end{aligned}$$

(6) the proposed ratio estimator $\bar{y}_{pmr}^{(8)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-3.73487\times10^{-5}<0\end{aligned}$$

(7) the proposed ratio estimator $\bar{y}_{pmr}^{(10)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\theta_1^{*2})C_{x_1}^2+(w_2^{*2}\alpha_2^2-\theta_2^{*2})C_{x_2}^2\\&-2(w_1^*\alpha_1-\theta_1^*)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2-\theta_2^*)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\theta_1^*\theta_2^*)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.000130956<0\end{aligned}$$

The result shows that the condition (14) is satisfied.

From Table 3, we understand that the efficiency of proposed ratio estimator $\bar{y}_{pmr}^{(10)}$ is as good as the estimator $\bar{y}_{r2}^{\gamma}$ proposed by Abu-Dayeh *et al.* We also understand that the proposed ratio estimators $\bar{y}_{pmr}^{(i)}$, $i=1,2,\cdots 10$ are more efficient than the estimator $\bar{y}_{r2}^{\varepsilon}$ proposed by Abu-Dayeh *et al.* When we examine the condition (16), for this data set, we see that all of them are satisfied as follows:

(8) the proposed ratio estimator $\bar{y}_{pmr}^{(1)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.102827011<0\end{aligned}$$

(9) the proposed ratio estimator $\bar{y}_{pmr}^{(2)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.02009932<0\end{aligned}$$

(10) the proposed ratio estimator $\bar{y}_{pmr}^{(3)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.02028071<0\end{aligned}$$

(11) the proposed ratio estimator $\bar{y}_{pmr}^{(4)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.017688305<0\end{aligned}$$

(12) the proposed ratio estimator $\bar{y}_{pmr}^{(5)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.02034572<0\end{aligned}$$

(13) the proposed ratio estimator $\bar{y}_{pmr}^{(6)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.020382497<0\end{aligned}$$

(14) the proposed ratio estimator $\bar{y}_{pmr}^{(7)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.020361337<0\end{aligned}$$

(15) the proposed ratio estimator $\bar{y}_{pmr}^{(8)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.020291996<0\end{aligned}$$

(16) the proposed ratio estimator $\bar{y}_{pmr}^{(9)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.019053895<0\end{aligned}$$

(17) the proposed ratio estimator $\bar{y}_{pmr}^{(10)}$

$$\begin{aligned}&(w_1^{*2}\alpha_1^2-\varepsilon_1^{*2}\gamma_1^2)C_{x_1}^2+(w_2^{*2}\alpha_2^2-\varepsilon_2^{*2}\gamma_2^2)C_{x_2}^2\\&-2(w_1^*\alpha_1+\varepsilon_1^*\gamma_1)\rho_{yx_1}C_yC_{x_1}-2(w_2^*\alpha_2+\varepsilon_2^*\gamma_2)\rho_{yx_2}C_yC_{x_2}\\&+2(w_1^*w_2^*\alpha_1\alpha_2-\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2)\rho_{x_1x_2}C_{x_1}C_{x_2}=-0.020385603<0\end{aligned}$$

From Table 3, we also understand that the most efficient estimator is the proposed ratio estimator $\bar{y}_{pmr}^{(10)}$ and the estimator $\bar{y}_{r2}^{\gamma}$ proposed by Abu-Dayeh *et al.* Therefore, we suggest that we should apply the proposed ratio estimator $\bar{y}_{pmr}^{(10)}$ and the estimator $\bar{y}_{r2}^{\gamma}$ proposed by Abu-Dayeh *et al.* to this data set.

Conclusions

We develop a class of ratio estimators of a finite population mean using two auxiliary variables and theoretically show that the proposed family ratio estimators are more efficient than the traditional ratio estimator and the estimators proposed by Abu-Dayeh *et al.* in certain conditions. These theoretical conditions are also satisfied by the results of a numerical example.

Author Contributions

Conceived and designed the experiments: JL. Performed the experiments: JL. Analyzed the data: JL ZY. Contributed reagents/materials/analysis tools: JL ZY. Wrote the paper: JL.

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